Test Series: CSIR NET/JRF Exam

Physical Sciences

Test Paper: Quantum Mechanics-I

Instructions: 1. Attempt all Questions. **Max Marks: 185**

2. There is a negative marking of 1/4 for each wrong answer.
3. Each Question in Section-A carry 3.5 marks.
4. Each Question in Section-B carry 5 marks.

Section- A

Question- 1: For the Hydrogen atom the radial part of the wave function is given as

$$R_{20}(r) = \frac{C_0}{2a}\left(1-\frac{r}{2a}\right)e^{-r/2a}$$

What is the wave function for the state ψ_{200} ?

(a) $\frac{1}{\sqrt{\pi}a^{3/2}}\left(1-\frac{r}{2a}\right)e^{-r/2a}$

(b) $\frac{1}{\sqrt{8\pi}a^{3/2}}\left(1-\frac{r}{2a}\right)e^{-r/2a}$

(c) $\frac{1}{a^{3/2}}\left(1-\frac{r}{a}\right)e^{-r/a}$

(d) $\frac{1}{\sqrt{2}a^{3/2}}\left(1+\frac{r}{2a}\right)e^{-r/2a}$

Question- 2: For the Hydrogen atom the radial part of the wave function is given as

$$R_{21}(r) = \frac{C_0 r}{4a^2}e^{-r/2a}$$

What is the wave function for the state ψ_{211} ?

(a) $-\frac{1}{8a^{5/2}} re^{-r/2a} \sin\theta \, e^{i\phi}$ (b) $\frac{1}{8a^{5/2}} re^{-r/2a} \sin\theta \, e^{i\phi}$

(c) $\frac{1}{8a^{5/2}} re^{-r/2a} \sin\theta \, e^{-i\phi}$ (d) $-\frac{1}{8a^{5/2}} re^{-r/2a} \sin\theta \, e^{-i\phi}$

Question- 3: Let ψ_{nlm} denote the eigen functions of a Hamiltonian for a spherically symmetric potential V(r). The wave function $\psi = \frac{1}{4}\left[\psi_{210} + \sqrt{5}\psi_{21-1} + \sqrt{10}\psi_{213}\right]$ is an eigen function only of

(a) H, L^2 and L_Z (b) H and L_Z (c) H and L^2 (d) L^2 and L_Z

Question- 4: If the ground state and 1st exited state of LHO,

$$\psi_0\langle x\rangle = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{\frac{1}{2}} e^{-\alpha^2 x^2/2}$$

$$\psi_1\langle x\rangle = \left(\frac{\alpha}{2\sqrt{\pi}}\right)^{\frac{1}{2}} e^{-\alpha^2 x^2/2}.\, 2ax \qquad \text{where } \alpha = \sqrt{\frac{m\omega}{\hbar}}$$

and the state of the function $\psi(x,t) = \frac{1}{\sqrt{2}}[\psi_0(x,t) + \psi_1(x,t)]$

What is the expectation value of energy of the LHO ?

(a) 1/2 ħw (b) ħw (c) 3/2 ħw (d) ħw/4

Question- 5: If the expectation value of the position square of harmonic oscillator is

$$< x^2 > = \frac{(n+\frac{1}{2})\hbar}{m\omega}.$$

What is the expectation value of potential energy ?

(a) E_n (b) $E_n/2$ (c) E_n^2 (d) $E_n^2/3$

Question- 6: Consider a state which is eigen state of $\hat{L}^2$ and $\hat{L}_z$

$$\hat{L}^2 \mid \psi\rangle = l(l+1)\,\hbar^2 \mid\psi\rangle, \qquad \hat{L}_z \mid \psi\rangle = m\hbar \mid \psi\rangle$$

What are the eigen values of $\langle \hat{L}_x \rangle$ and $\langle \hat{L}_x^2 \rangle$ respectively for this state?

(a) $\hbar$, $\frac{1}{2}[l(l+1)-m^2]\hbar^2$ (b) $\frac{3}{2}[l(l+1)-m^2]\hbar^2, 0$

(c) 0, $\frac{1}{2}[l(l+1)-m^2]\hbar^2$ (d) $2\hbar$, $\frac{1}{2}[l(l+1)-m^2]\hbar^2$

Question-7: In the above problem (Qs.6), the value of $\Delta\hat{L}_x$ is

(a) 0 (b) $\sqrt{\hbar}$ (c) $\sqrt{\frac{1}{2}[l(l+1)-m^2]}\,\hbar$ (d) $\sqrt{l(l+1)-m^2}\,\hbar$

Question-8: A quantum mechanical system's state of a particle, with Cartesian coordinates x,y,z is described by the normalised wave function

$$\psi(x,y,z) = \frac{\alpha^{5/2}}{\sqrt{\pi}} z e^{-\left[\alpha(x^2+y^2+z^2)^{1/2}\right]}$$

What are the values of $\hat{L}^2$ and $\hat{L}_z$ respectively associated with the state?

(a) $\hbar^2$, 1 (b) $\hbar^2$, 0 (c) 0, $\hbar^2$ (d) $2\hbar^2$, 0

Question- 9: Consider the state $|\psi\rangle = \frac{1}{3}[|11\rangle + |10\rangle + |1-1\rangle]$, where first number in each term represent l and second term m. What is the value of $\langle \hat{L}_+ \rangle$?

(a) $\frac{\sqrt{2}}{3}\hbar$ (b) $\frac{2\sqrt{2}}{3}\hbar$ (c) $\sqrt{2}\,\hbar$ (d) $\frac{\hbar}{3}$

Question- 10: The wave function of a state of hydrogen atom is given by

$\psi = \psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\,\psi_{21-1}$, where ψ_{nlm} denotes the eigen states with quantum numbers n, l, m in the usual notation. The expectation value of $\hat{L}^2$ and $\hat{L}_z$ respectively in the state ψ is

(a) $\frac{15}{8}\hbar^2, \frac{\hbar}{8}$ (b) $\frac{3}{2}\hbar^2, \frac{\hbar}{8}$ (c) $\frac{3\hbar^2}{8}, \frac{3\hbar}{8}$ (d) $\frac{\hbar^2}{8}, \frac{\hbar}{8}$

Section- B

Question-11: Consider a system whose state is given in terms of an orthogonal set of three vectors $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle$

$$|\psi\rangle = A\left[\sqrt{3}\,|\phi_1\rangle + 2\,|\phi_2\rangle + \sqrt{2}\,|\phi_3\rangle\right]$$

What are the probabilities of finding the system in state $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle$ respectively?

(a) $\frac{4}{9}, \frac{1}{3}, \frac{2}{9}$ (b) $\frac{2}{9}, \frac{4}{9}, \frac{1}{3}$ (c) $\frac{1}{2}, \frac{2}{3}, \frac{1}{3}$ (d) $\frac{1}{3}, \frac{4}{9}, \frac{2}{9}$

Question-12: Consider a system whose state is given in terms of complete and orthnormal set of 5 vectors i.e. $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle, |\phi_4\rangle, |\phi_5\rangle$ as follows

$$|\psi\rangle = \frac{1}{\sqrt{19}}|\phi_1\rangle + \frac{2}{\sqrt{19}}|\phi_2\rangle + \sqrt{\frac{2}{19}}\,|\phi_3\rangle, + \sqrt{\frac{3}{19}}\,|\phi_4\rangle + \sqrt{\frac{5}{19}}\,|\phi_5\rangle,$$

where $|\phi_n\rangle$ are the eigen states to the system's Hamiltonian

$H\,|\phi_n\rangle = nE_0\,|\phi_n\rangle$ with, $n = 1,2,3,4,5$

where E_0 has the dimension of energy.

If we measure the energy then what is the probability of getting result $3E_0$?

(a) $\frac{1}{15}$ (b) $\frac{4}{15}$ (c) $\frac{2}{15}$ (d) $\frac{1}{5}$

Question- 13: In above problem (Qs.12), what is the average energy of one such system ?

(a) 15/52 ε_0 (b) 10/52 ε_0 (c) 52/10 ε_0 (d) 52/15 ε_0

Question- 14: A particle of mass m which moves freely inside infinite potential well of length a has the following initial wave function at t = 0

$$\psi(x,0) = \left[\frac{A}{\sqrt{a}}\sin\frac{\Pi x}{a} + \sqrt{\frac{3}{5a}}\sin\frac{3\Pi x}{a} + \sqrt{\frac{1}{5a}}\sin\frac{5\Pi x}{a}\right]$$

where A is real constant.

If measurements of the energy are carried out, different values will be found for different states with different corresponding probabilities. What is the average energy of the system ?

(a) $\frac{29}{5}\left(\frac{\pi^2\hbar^2}{2ma^2}\right)$ (b) $\frac{5}{29}\left(\frac{\pi^2\hbar^2}{2ma^2}\right)$ (c) $\frac{58}{10}\left(\frac{\pi^2\hbar^2}{ma^2}\right)$ (d) $\frac{58}{5}\left(\frac{\pi^2\hbar^2}{ma^2}\right)$

Question- 15: In above problem (Qs.14), what is the probability of finding the system at a time t in the state $\phi(x,t) = \sqrt{\frac{2}{a}}\sin\frac{5\Pi x}{a}e^{-iE_5t/\hbar}$?

(a) 1/10 (b) 1/5 (c) 2/5 (d) 3/10

Question- 16: In above problem (Qs.14), what is the probability of finding the system at a time t in the state $\chi(x,t) = \sqrt{\frac{2}{a}}\sin\frac{2\Pi x}{a}e^{-iE_2t/\hbar}$?

(a) 1 (b) 0 (c) ½ (d) 1/5

Question- 17: Consider a spin 1/2 particle in the presence of homogeneous magnetic field of magnitude B along z-axis which is prepared initially in a state $|\psi> = \frac{1}{\sqrt{2}}(|\downarrow> + |\uparrow>)$ at time t = 0. At what time t will the particle be in the state -|ψ> (μ_B is Bohr magnitude)?

(a) $t = \pi\hbar/\mu_B B$ (b) $t = 2\pi\hbar/\mu_B B$ (c) $t = \pi\hbar/2\mu_B B$ (d) Never

Question- 18: A particle of mass m, which moves freely inside an infinite potential well of length a, is initially in the stat $\psi(x,0) = \left[\sqrt{\frac{3}{5a}}\sin\frac{3\Pi x}{a} + \sqrt{\frac{1}{5a}}\sin\frac{5\Pi x}{a}\right]$

What is ψ(x, t) at any latter time t ?

(a) $\sqrt{\frac{3}{5}}\phi_3(x)e^{-\frac{iE_3t}{\hbar}} + \sqrt{\frac{1}{5}}\phi_5(x)e^{-iE_5t/\hbar}$

(b) $\sqrt{\frac{3}{10}}\phi_3(x)e^{-\frac{iE_3t}{\hbar}} + \sqrt{\frac{1}{10}}\phi_5(x)e^{-iE_5t/\hbar}$

(c) $\sqrt{\frac{3}{10}}\phi_3(x)e^{-\frac{iE_5t}{\hbar}} + \sqrt{\frac{1}{10}}\phi_5(x)e^{-iE_3t/\hbar}$

(d) $\sqrt{\frac{3}{5}}\phi_3(x)e^{-\frac{iE_1t}{\hbar}} + \sqrt{\frac{1}{10}}\phi_5(x)e^{-iE_2t/\hbar}$

Question- 19: In above problem (Qs.18), what is the probability current density?

(a) $\frac{3}{10}\phi_3{}^2(x) + \frac{\sqrt{3}}{5}\phi_3(x)\phi_5(x)\cos\frac{16E1t}{\hbar} + \frac{1}{10}\phi_5{}^2(x)$

(b) $\frac{3}{10}\phi_3{}^2(x) + \frac{1}{10}\phi_5{}^2(x)$

(c) $\frac{1}{10}\phi_3{}^2(x) + \frac{3}{10}\phi_5{}^2(x)$

(d) $\frac{3}{10}\phi_3{}^2(x) - \frac{1}{10}\phi_5{}^2(x)$

Question- 20: A system is initially in the state

$$|\psi_0\rangle = A\left[\sqrt{2}\,|\,\phi_1\rangle + \sqrt{3}\,|\,\phi_2\rangle + |\,\phi_3\rangle + |\,\phi_4\rangle\right]$$

where $|\phi_n>$ are eigen state of the system's Hamiltonian such that

$$\hat{H}\,|\,\phi_n\rangle = n^2\varepsilon_0\,|\,\phi_n\rangle$$

If energy is measured, what is the probability of getting result $4\varepsilon_0$?

(a) 2/7 (b) 3/7 (c) 1/7 (d) 4/7

Question- 21: In above problem (Qs.20), consider an operator $\hat{A}$ whose action on $|\phi_n>$ is defined by $\hat{A}\,|\,\phi_n\rangle = (n+1)a_0\,|\,\phi_n\rangle$. If A is measured, what is the probability of getting result $4a_0$?

(a) 2/7 (b) 3/7 (c) 1/7 (d) 4/7

Question- 22: In above problem (Qs.20), suppose that a measurement of the energy yield $4\varepsilon_0$, If we measure A immediately afterwards, what value will be obtained?

(a) a_0 (b) $2a_0$ (c) $3a_0$ (d) 0

Question- 23: Consider a physical system whose Hamiltonian $\hat{H}$ and initial state

$$\hat{H} = \varepsilon_0 \begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}; \mid \psi_0\rangle = \frac{1}{\sqrt{5}} \begin{pmatrix} 1-i \\ 1-i \\ 1 \end{pmatrix}$$

where ε_0 has the dimension of energy.

If energy is measured, what is the probability of getting result $-\varepsilon_0$ for degenerate state $E_2 = E_3 = -\varepsilon_0$?

(a) 2/5 (b) 1/5 (c) 3/5 (d) 4/5

Question- 24: In above problem (Qs.23), what is the expectation value of the Hamiltonian H?

(a) $1/5\ \varepsilon_0$ (b) $-1/5\ \varepsilon_0$ (c) $2/5\ \varepsilon_0$ (d) $-2/5\ \varepsilon_0$

Question- 25: Consider a system whose Hamiltonian H and operator A are given by

$$\hat{H} = \varepsilon_0 \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}; \ \hat{A} = a \begin{pmatrix} 0 & 4 & 0 \\ 4 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

where ε_0 has the dimension of energy

Suppose that when we measure the energy, we obtain the energy $-\varepsilon_0$, immediately after wards, we measure A. What is the probability of getting result 0?

(a)16/17 (b) 1/34 (c) 1/17 (d) 17/16

Question- 26: In above problem (Qs.25), what is the uncertainty ΔA?

(a) a/2 (b) a^2 (c) $2a^2$ (d) a

Question- 27: At time t = 0 the wave function for hydrogen atom is

$$\psi(r,0) = \frac{1}{\sqrt{10}}\left(2\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1}\right)$$

where the subscripts are values of the quantum numbers n, l, m. Ignore the spin and radiative transitions.

What is the expectation value for the energy of this system in units of $\frac{mc^2}{2}\left(\frac{e^2}{\hbar c}\right)^2$?

(a) –0.55 (b) –0.55/2 (c) +0.55 (d) +0.55/2

Question- 28: In above problem (Qs.27), what is the probability of finding the system with, $n = 2$, $l = 1$, $m = +1$, as a function of time?

(a) $1/\sqrt{5}$ (b) 0 (c) 1/5 (d) 2/5

Question- 29: A particle of mass m is confined to a 1-dimension region $0 \le x \le a$. At $t = 0$ its normalized wave function is

$$\psi(x, t=0) = \sqrt{\frac{8}{5a}}\left[1+\cos\left(\frac{\pi x}{a}\right)\right]\sin\left(\frac{\pi x}{a}\right)$$

What is the average energy of the system at $t = 0$ and at $t = t_0$?

(a) $\pi^2\hbar^2/ma^2$ (b) $\pi^2\hbar^2/5ma^2$ (c) $2\pi^2\hbar^2/5ma^2$ (d) $4\pi^2\hbar^2/5ma^2$

Question- 30: In above problem (Qs.29), what is the probability that the particle is found in the left of the box (i.e., in the region $0 \le x \le a/2$) at $t = t_0$?

(a) $\frac{1}{2}+\frac{16}{15}\cos\left(\frac{3\pi^2\hbar t_0}{2ma^2}\right)$ (b) $\frac{1}{2}-\frac{16}{15}\cos\left(\frac{3\pi^2\hbar t_0}{2ma^2}\right)$ (c) $\frac{1}{2}+\frac{16}{15}\sin\left(\frac{3\pi^2\hbar t_0}{2ma^2}\right)$ (d) $\frac{1}{2}-\frac{16}{15}\sin\left(\frac{3\pi^2\hbar t_0}{2ma^2}\right)$

Question- 31: A particle of mass m is confined in a one dimensional infinite well extending from $x = 0$ to $x = +L$. The particle is in its ground state. The walls of the box are moved suddenly to form a box extending from $x = 0$ to $x = 4L$. What is the probability that the particle will be in the ground state after this sudden expansion?

(a) $64/(3\pi)^2$ (b) $16/(3\pi)^2$ (c) $128/(15\pi)^2$ (d) $8/(3\pi)^2$

Question- 32: A particle in the ground state of an infinite square well potential is given by

$$V(x) = \begin{cases} 0 \;\; for -\frac{a}{2} \le x \le \frac{a}{2} \\ \infty, \qquad otherwise \end{cases}$$

The probability to find the particle in the interval –a/4 to +a/4 is

(a) $1/2 + 1/\pi$ (b) $1/2 - 1/\pi$ (c) $1 + 2/\pi$ (d) $1 - 2/\pi$

Question- 33: A quantum mechanical particle in a harmonic oscillator has the initial wave function $\psi_0(x) + \psi_1(x)$, where ψ_0 and ψ_1 are the real wave function in ground state and first excited state of the harmonic oscillator Hamiltonian. For convenience choose $m = \hbar = w = 1$ for the oscillator. What is the probability density of finding the particle at any time $t = \pi$?

(a) $[\psi_1(x) - \psi_0(x)]^2$ (b) $[[\psi_1(x)]^2 - [\psi_0(x)]^2$

(c) $[\psi_1(x) + \psi_0(x)]^2$ (d) $[\psi_1(x)]^2 + [\psi_0(x)]^2$

Question- 34: A spin-1 particle is in the state $|\psi>$ described by the column matrix

$|\psi\rangle = \frac{1}{\sqrt{10}}\begin{pmatrix} 2 \\ \sqrt{2} \\ 2i \end{pmatrix}$ in the S_z basis.

What is the probability that a measurement of operator S_z will yield the result $+\hbar$ for the state $S_x|\psi>$ is ?

(a) 1/2 (b) $1/\sqrt{3}$ (c) 1/4 (d) 1/6

Question- 35: If x(t) be the position operator at a time t in the Heisenberg picture for a particle described by the Hamiltonian, $\hat{H} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$, what is the value of $e^{i\omega t}\langle 0|\hat{x}(t)\hat{x}(0)|0\rangle$, where

|0> is the ground state?

(a) ħ/2mw (b) ħ/4mw (c) ħ/mw (d) 2 ħ/mw

Question- 36: Suppose a spin $\frac{1}{2}$ particle is in the state

$$\chi = \frac{1}{\sqrt{11}}\begin{pmatrix} 1+i \\ 3 \end{pmatrix}$$

If we measure S_z, what is the probability of getting $+\frac{\hbar}{2}$ *and* $-\frac{\hbar}{2}$ respectively?

(a) $\frac{2}{11}, \frac{9}{11}$ (b) $\frac{9}{11}, \frac{2}{11}$ (c) $\frac{1}{11}, \frac{10}{11}$ (d) $\frac{3}{11}, \frac{8}{11}$

Question- 37: In the above (Qs.36) what is the expectation value of S_x?

(a) $\frac{\hbar}{11}$ (b) $\frac{2\hbar}{11}$ (c) $3\hbar$ (d) $\frac{3\hbar}{11}$

Question- 38: An electron is in the spin state

$$\chi = A\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

What is the probability of getting $+\frac{\hbar}{2}$, if we measure S_x?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

Question- 39: In the above problem (Qs.38), what is the probability of getting $-\frac{\hbar}{2}$ if we measure S_y?

(a) $\frac{1}{25}$ (b) $\frac{49}{50}$ (c) $\frac{1}{50}$ (d) $\frac{24}{25}$

Question- 40: In the above (Qs.38), what is the expectation value of S_z?

(a) $-\frac{7\hbar}{20}$ (b) $+\frac{7\hbar}{20}$ (c) $-\frac{7\hbar}{50}$ (d) $+\frac{7\hbar}{50}$