Scattering Theory (Quantum Mechanics)

Om's Scattering Formula:

Scattering amplitude f(θ):

Potential V(r)= $\beta\, e^{-\mu r}\, r^n$, where n= -1, 0,1,2

For lower value of n (i.e. n= -1, 0)

$$f(\theta)= \frac{C_0}{(\mu^2+q^2)^{n+2}} \quad \text{.........(1)}$$

For higher value of n (i.e. n= 1, 2)

$$f(\theta)= \frac{C_0}{(\mu^2+q^2)^{n+2}} \frac{1}{q^{n-1}} [C_1 q^{n-1}\, \mu^{n+1} - C_2 q^{n+1}\, \mu^{n-1}] \quad \text{........(2)}$$

where C_0, C_1, C_2 are constants & q is momentum transfer.

Problems:

Prob.1: For Yukawa potential V(r)= $\frac{\beta\, e^{-\mu r}}{r}$, f(θ)= ? [NET June 2013]

Sol: V(r)= $\frac{\beta\, e^{-\mu r}}{r} = \beta\, e^{-\mu r}\, r^{-1}$ => n= -1

Using Om's scattering formula for scattering amplitude (for lower values of n)

$$f(\theta)= \frac{C_0}{(\mu^2+q^2)^{n+2}},\ n= -1$$

So, $f(\theta)= \frac{C_0}{(\mu^2+q^2)^{-1+2}} = \frac{C_0}{(\mu^2+q^2)}$ Ans.

Prob.2: For potential $V(r) = \beta\, e^{-\mu r}$, $f(\theta)$= ? [NET June 2014]

Sol: $V(r) = \beta\, e^{-\mu r} = \beta\, e^{-\mu r}\, r^0$ => n= 0

Using Om's scattering formula for scattering amplitude (for lower values of n)

$f(\theta)= \frac{C_0}{(\mu^2+q^2)^{n+2}}$, n= 0

So, $f(\theta)= \frac{C_0}{(\mu^2+q^2)^2}$ Ans.

Prob.3: For potential $V(r) = \beta\, e^{-\mu r}$ r , $\sigma(\theta)$= ? [NET Dec 2016]

Sol: $V(r) = \beta\, e^{-\mu r}\, r = \beta\, e^{-\mu r}\, r^1$ => n=1

Using Om's scattering formula for scattering amplitude (for higher values of n)

$f(\theta)= \frac{C_0}{(\mu^2+q^2)^{n+2}} \frac{1}{q^{n-1}} [C_1 q^{n-1}\, \mu^{n+1} - C_2 q^{n+1}\, \mu^{n-1}]$, n=1

So, $f(\theta)= \frac{C_0}{(\mu^2+q^2)^{1+2}} \frac{1}{q^{1-1}} [C_1 q^{1-1}\, \mu^{1+1} - C_2 q^{1+1}\, \mu^{1-1}]$

$f(\theta) = \frac{C_0}{(\mu^2+q^2)^3} [C_1\, \mu^2 - C_2 q^2]$

$\sigma(\theta) = |f(\theta)|^2$

$= \frac{{C_0}^2}{(\mu^2+q^2)^6} [C_1\, \mu^2 - C_2 q^2]^2 = \frac{{C_0}^2(-C_2 q^2)^2}{q^{12}\,(1+\frac{\mu^2}{q^2})^6} [1 - \frac{C_1\, \mu^2}{C_2 q^2}]^2$

Since q>> μ as given in Qs. So the terms $\frac{C_1\, \mu^2}{C_2 q^2}$ & $\frac{\mu^2}{q^2}$ can be neglected.

So, $\sigma(\theta) = \frac{C_0{}^2 C_2{}^2 q^4}{q^{12}} = \frac{C_0{}^2 C_2{}^2}{q^8}$

Or $\sigma(\theta)\ \alpha\ \frac{1}{q^8}\ \alpha\ q^{-8}$ Ans.

Prob.4: For potential V(r) = $\beta\, e^{-\mu r}\, r^2$, f(θ)= ? [Qs of Future]

Sol: V(r) = $\beta\, e^{-\mu r}\, r^2$ => n=2

Using Om's scattering formula for scattering amplitude (for higher values of n)

$$f(\theta)= \frac{C_0}{(\mu^2+q^2)^{n+2}} \frac{1}{q^{n-1}} [C_1 q^{n-1}\, \mu^{n+1} - C_2 q^{n+1}\, \mu^{n-1}],\ n=2$$

$$\text{So, } f(\theta)= \frac{C_0}{(\mu^2+q^2)^{2+2}} \frac{1}{q^{2-1}} [C_1 q^{2-1}\, \mu^{2+1} - C_2 q^{2+1}\, \mu^{2-1}]$$

$$= \frac{C_0}{(\mu^2+q^2)^4} \frac{1}{q} [C_1 q^1\, \mu^3 - C_2 q^3\, \mu^1]$$

$$f(\theta) = \frac{C_0}{(\mu^2+q^2)^4} [C_1\, \mu^3 - C_2 q^2\, \mu^1] \quad \text{Ans.}$$

Perturbation Theory

First Order Energy Correction $E_n^{(1)}$:

Total Hamiltonian H=$H_0 + H_P$

Where H_0 is total Hamiltonian, H_P is Perturbed Hamiltonian.

$$E_n^{(1)} = <\Psi_n|H_P|\Psi_n> \quad \ldots\ldots\ldots\ldots..(1)$$

Trick:

$$E_n^{(1)} = \text{Perturbed Potential} \times \frac{\text{Perturbed Area}}{\text{Total Area}} \quad \ldots.(2)$$

Where Total Area= Perturbed Pot.× Width of Well

Perturbed Area= Perturbed Pot.× Perturbed width

Or $$E_n^{(1)} = \text{Perturbed Potential} \times \frac{\text{Perturbed width}}{\text{Total width of well}} \quad \ldots.(3)$$

Note: 1.This trick is valid when the perturbed width lies on the node or antinode of the Infinite square well potential.

2. If the perturbed width doesn't lie at the node or antinode of the Infinite square well potential then the First order energy correction is given by adding a constant term in the result observed from above trick.

Problems:

Prob.1:

The unperturbed wave function for the infinite square well potential is given by $\Psi_n^{(o)} = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$. Suppose we perturb the system by simply raising the floor of the well by constant amount V_0 only half way across the well. What is the first order energy correction for nth state? [GATE 2015]

Sol: Here Perturbed potential $H_P = V_0$

$$E_n^{(1)} = V_0 \frac{a/2}{a} = \frac{V_0}{2}$$ Ans.

Prob.2:

The unperturbed wave function for the infinite square well potential is given by $\Psi_n^{(o)} = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$. Suppose we perturb the system by simply lowering the floor of the well by constant amount V_0 only half way across the well. What is the first order energy correction for nth state?

Sol: In case of lowering the floor, the perturbed potential $H_P = -V_0$

$$E_n^{(1)} = -V_0 \frac{a/2}{a} = -\frac{V_0}{2}$$ Ans.